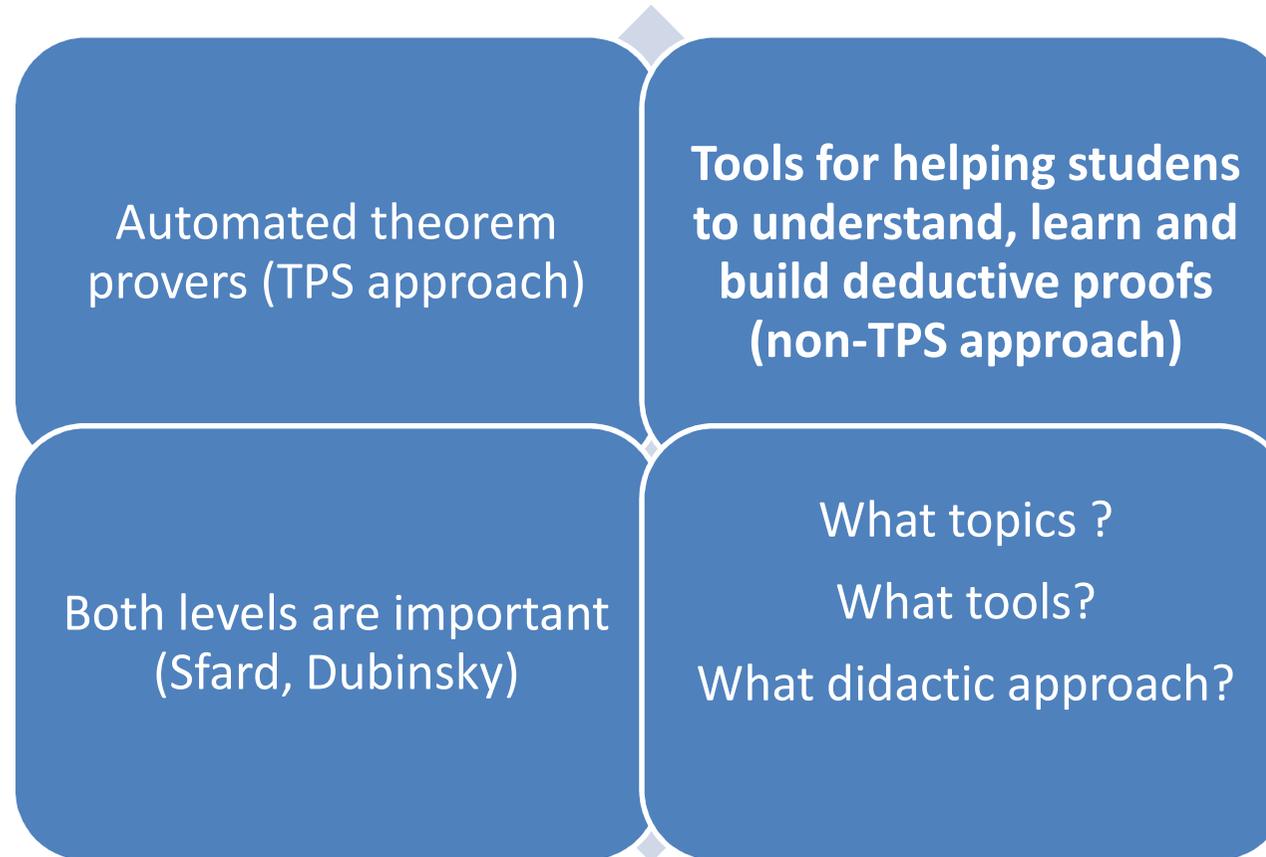


Technology as a support for generating and presenting proofs in geometry

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Technology and proving in school geometry?



Technology and non TPS proving

- **Observation of facts**
- **Building/writing up proofs**
- **Validating proofs**
- **Presenting proofs**
- **Learning to prove facts**

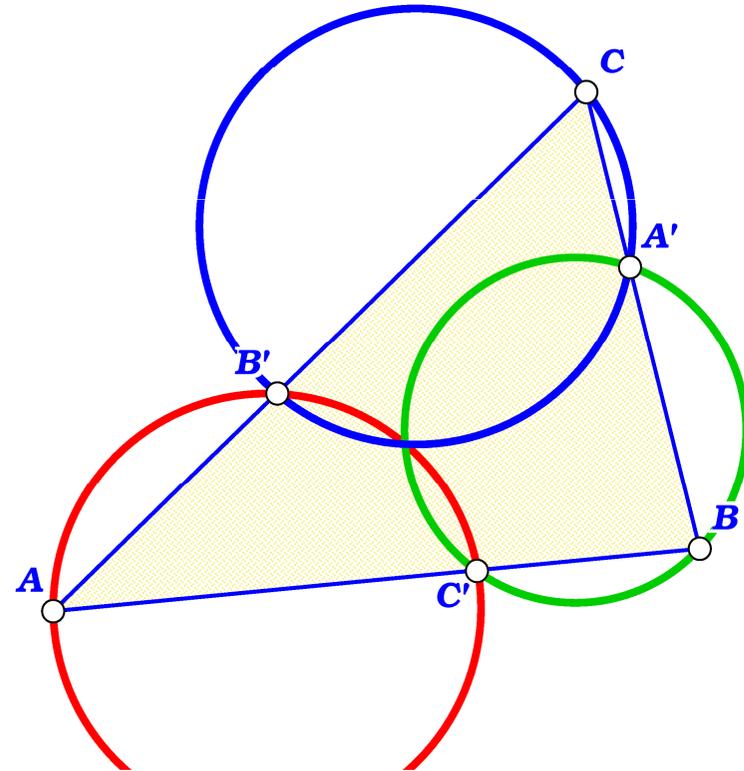
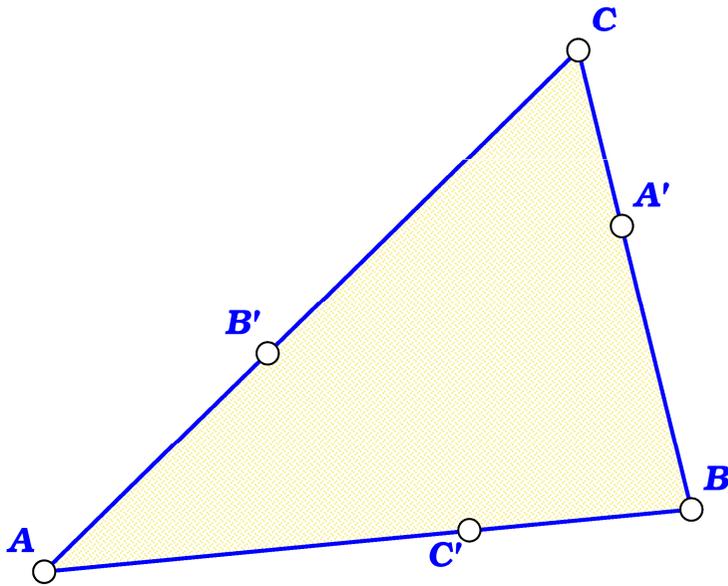
OK Geometry

- **OK Geometry** is an **observational tool**. It detects (highly probable) properties of dynamic constructions (eg. from GeoGebra, Cinderella).
- With **OK Geometry** one can **generate hypothesis** and **proof-oriented exercises**.
- **OK Geometry** does not prove theorems, but may help students in building up proofs. Students need to relate the observations and provide the arguments.
- **OK Geometry** is free.
See <http://z-maga.si/index?action=article&id=40>

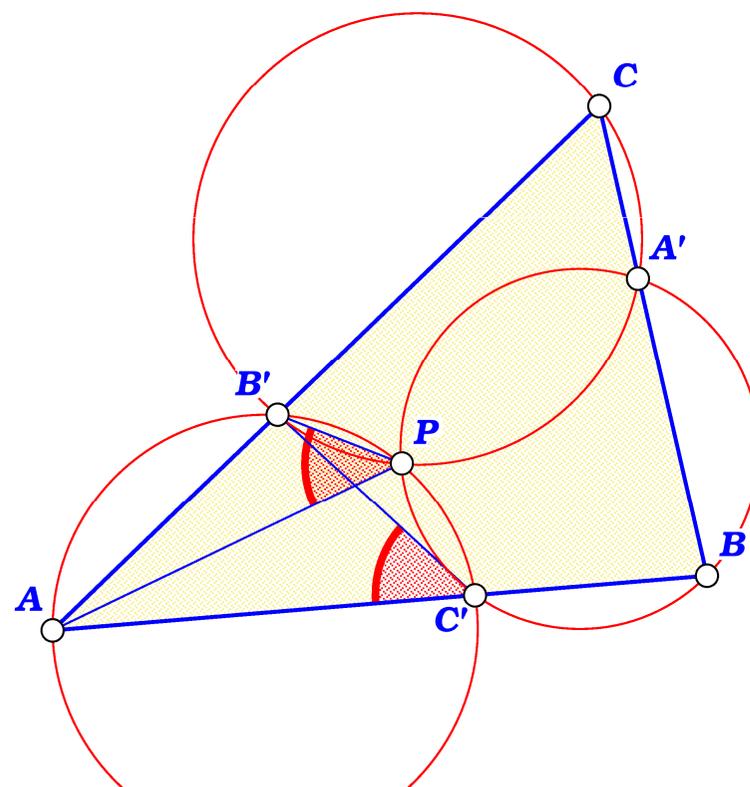
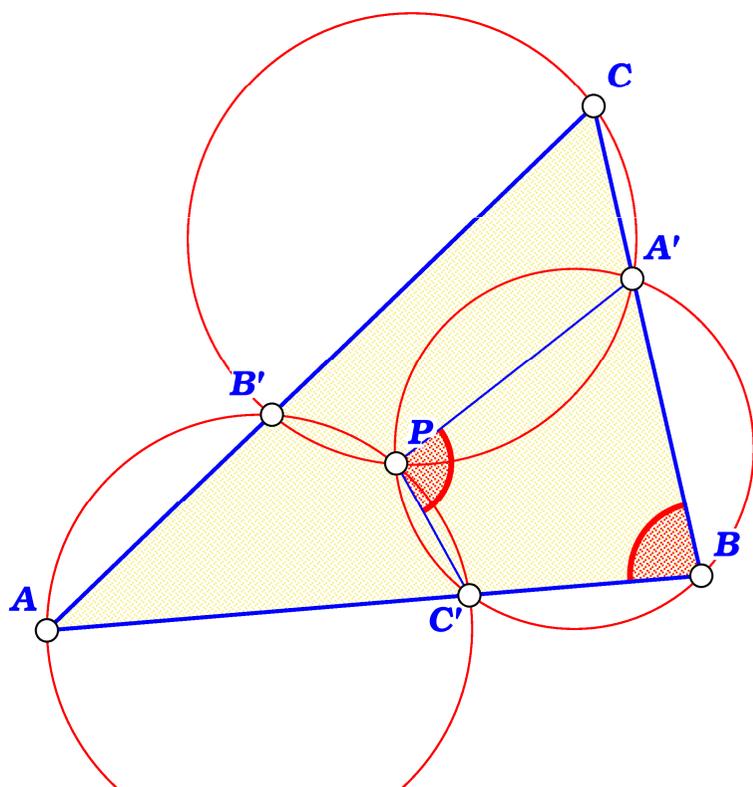
Basic approach of OK Geometry

On each side of a triangle there is a point. Observations?

An OK observation: The three circles meet in a common point.



One selects observations that may possibly be of use in proving the three circles claim.



The selected properties are collected and organised into a proof ...

OK Geometry Plus: Miquel0.pro

File Configure Commands Help

Task Sketch Observe **Project** Report

△ < > ↶ ↷ 5

Congruent angles

This is true because $\angle BPA$ and $B'CA$ are angles over the same arc AB' of the circle through $AB'C'$.

Observed properties

- Figure 1
Given is a triangle $\triangle ABC$. Let A', B', C' be any points on its sides BC, CA, AB .
- Figure 4
Given is a triangle $\triangle ABC$. Let A', B', C' be any points on its sides BC, CA, AB . The circles through $AB'C', A'B'C, A'B'C$ pass through a common point.
- Figure 5
- Congruent angles
This is true because $\angle BPA$ and $B'CA$ are angles over the same arc AB' of the circle through $AB'C'$.

Icon size Emphasize Bleach Highlight Labels Angles Other

Figure 1

Congruent angles

Figure 5

Figure 4

Zlatan Magajna, GADGME 2016, TarguMures

... and deductive argumentations are provided

The screenshot displays the OK Geometry Plus software interface. The main window shows a geometric construction of a triangle ABC with its Miquel point P . Three circles are drawn through the vertices and the Miquel point: $(AB'C')$, $(A'BC)$, and $(A'B'C)$. The Miquel point P is the intersection of the three circles. The main diagram is labeled '1' in the top toolbar. An 'Icon editor' dialog box is open, showing the 'Property' 'Congruent angles' and a 'Comment' that reads: 'This is true because $\angle B'PA$ and $\angle B'C'A$ are angles over the same arc AB' of the circle through $AB'C'$.' The dialog also has buttons for 'Emphasize', 'Labels', and 'Angles'. Three smaller inset diagrams are visible: 'Figure 4' (bottom center) shows a different view of the construction; 'Figure 5' (bottom right) shows the Miquel point P and the circles; and 'Congruent angles' (top right) shows the angles $\angle B'PA$ and $\angle B'C'A$ highlighted in red.

The obtained proof in paragraph form

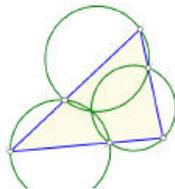
OK Geometry Report : Miquel theorem : (Miquel_hyper_proof.pro); 2016.8.25

Miquel theorem

1 Miquel theorem

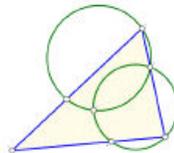
Given is a triangle $\triangle ABC$. Let A' , B' , and C' be arbitrary points on the sides BC , CA , and AB . The three circles through A, B', C' , through A', B, C' , and through A', B', C always meet in a common point.

Comment:



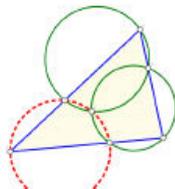
2 Strategy of the proof

Let P be the intersection other than A' of the circles through B, C, A' and through C, A, B' . We shall prove that P lies on the circle through A, B', C' .



3.0 Idea of the proof

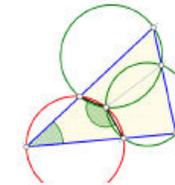
We shall prove that A, C', P, B' are cocyclic, i.e. that $AC'PB'$ is a cyclic quadrilateral. To prove this we shall use the theorem. A quadrilateral is cyclic if and only if its non adjacent angles are supplementary.



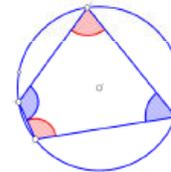
OK Geometry Report : Miquel theorem : (Miquel_hyper_proof.pro); 2016.8.25

3.1 Theorem

A quadrilateral $ABCD$ is cyclic if and only if its non-adjacent angles are supplementary.



3.2 Proof \Rightarrow



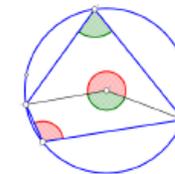
(\Rightarrow) Let $ABCD$ be a cyclic quadrilateral. Thus $ABCD$ is inscribed in a circle, let its centre be S .

Consider the opposite angles $\angle A$ and $\angle C$. These angles are related to complementary arcs BCD and DAB . The related complementary central angles $\angle BSD$ and $\angle DSB$ add up to 360° . Since the central angle of an arc is always twice the related arc angle, the angles $\angle A$ and $\angle C$ add up to 180° .

3.3 Proof \Leftarrow

(\Leftarrow) Assume now that in the quadrilateral $ABCD$ the angles $\angle A$ and $\angle C$ are supplementary. We claim that $ABCD$ is cyclic. Consider the circle k through A, B , and D . Assume, by contradiction, that C does not lay on k , but, for example, inside it. Let C' be the intersection, other than D , of the line BC with the circle k . Since $ABCD$ is cyclic, the angle $\angle C'$ is congruent to $\angle DCB$, since both are supplementary to $\angle A$. But this is impossible since in the triangle $\triangle CCB$ the exterior angle at C cannot be congruent to the opposite interior angle.

An analogous argument holds in case C lays outside the circle k .

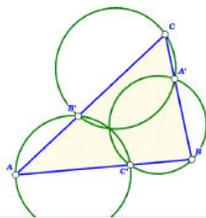


The obtained proof in two column form

OK Geometry Report : Miquel theorem; : (Miquel_hyper_proofpro); 2016.8.25

Miquel theorem

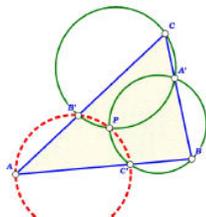
1 Miquel theorem



Given is a triangle $\triangle ABC$. Let A' , B' , and C' be arbitrary points on the sides BC , CA , and AB . The three circles through A, B', C' , through B, C', A' , and through C, A', B' all always meet in a common point.

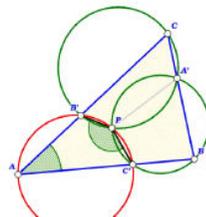
Comment:

2 Strategy of the proof



Let P be the intersection other than A' of the circles through B, C', A' and through C, A', B' . We shall prove that P lies on the circle through A, B', C' .

3.0 Idea of the proof



We shall prove that A, C', P, B' are cocyclic, i.e. that $AC'PB'$ is a cyclic quadrilateral. To prove this we shall use the theorem: A quadrilateral is cyclic if and only if its non adjacent angles are supplementary.

OK Geometry Report : Miquel theorem; : (Miquel_hyper_proofpro); 2016.8.25

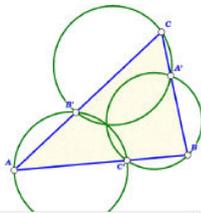
	<h3>3.1 Theorem</h3>	<p>A quadrilateral $ABCD$ is cyclic if and only if its non-adjacent angles are supplementary.</p>
	<h3>3.2 Proof \Rightarrow</h3>	<p>(\Rightarrow) Let $ABCD$ be a cyclic quadrilateral. Thus $ABCD$ is inscribed in a circle, let its centre be S. Consider the opposite angles $\angle A$ and $\angle C$. These angles are related to complementary arcs BCD and DAB. The related complementary central angles $\angle BSD$ and $\angle DSB$ add up to 360°. Since the central angle of an arc is always twice the related arc angle, the angles $\angle A$ and $\angle C$ add up to 180°.</p>

Other forms of proof presentation

OK Geometry Report : Miquel theorem; zm; (Miquel_hyper_proof/pro); 2016.8.31

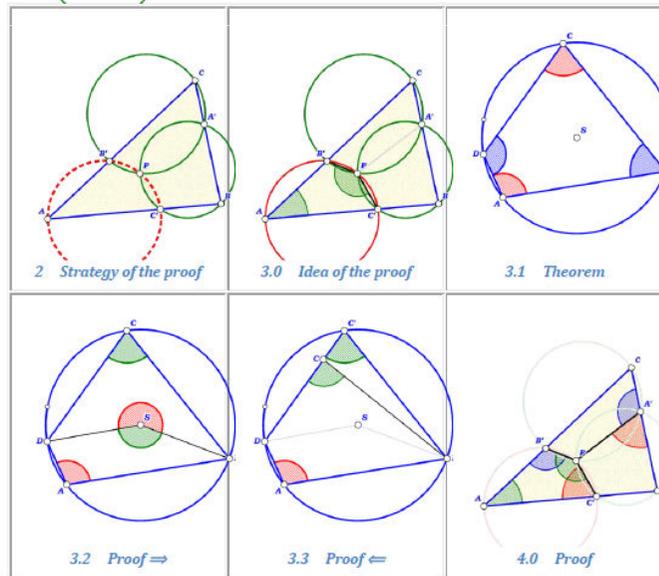
Miquel theorem

1 Miquel theorem

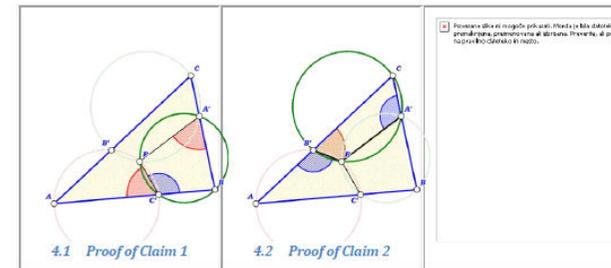


Given is a triangle $\triangle ABC$. Let A' , B' , and C' be arbitrary points on the sides BC , CA , and AB . The three circles through A, B', C' , through A', B, C' , and through A', B', C allways meet in a common point.

Comment:



OK Geometry Report : Miquel theorem; zm; (Miquel_hyper_proof/pro); 2016.8.31



2 Strategy of the proof

Let P be the intersection other than A' of the circles through B, C, A' and through C, A', B' . We shall prove that P lays on the circle through A, B', C' .

3.0 Idea of the proof

We shall prove that A, C', P, B' are cocyclic, ie. that $AC'PB'$ is a cyclic quadrilateral. To prove this we shall use the theorem:

A quadrilateral is cyclic if and only if its non adjacent angles are supplementary.

3.1 Theorem

A quadrilateral $ABCD$ is cyclic if and only if its non-adjacent angles are supplementary.

3.2 Proof \Rightarrow

(\Rightarrow) Let $ABCD$ be a cyclic quadrilateral. Thus $ABCD$ is inscribed in a circle, let its centre be S .

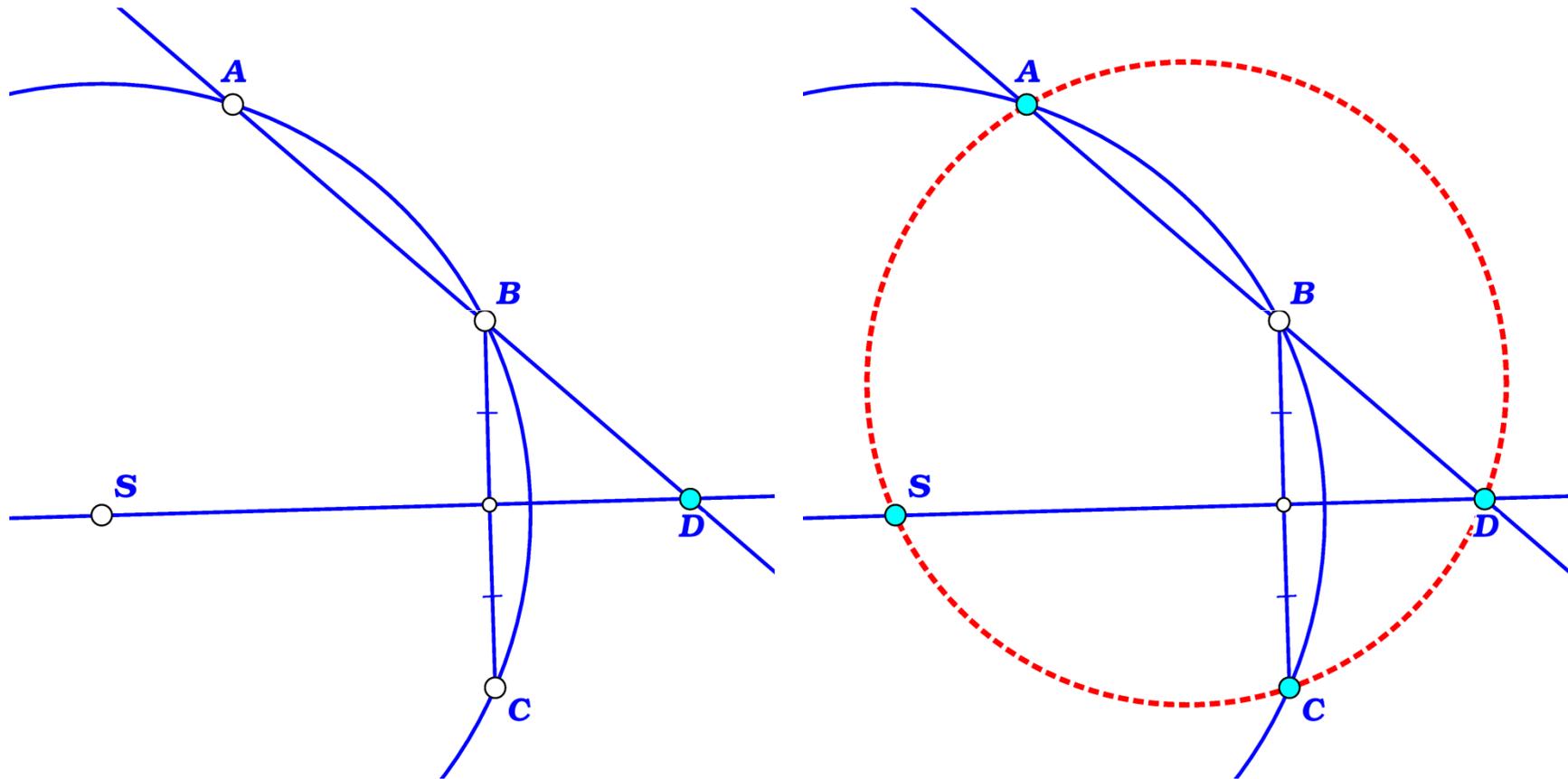
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3.3 Proof \Leftarrow

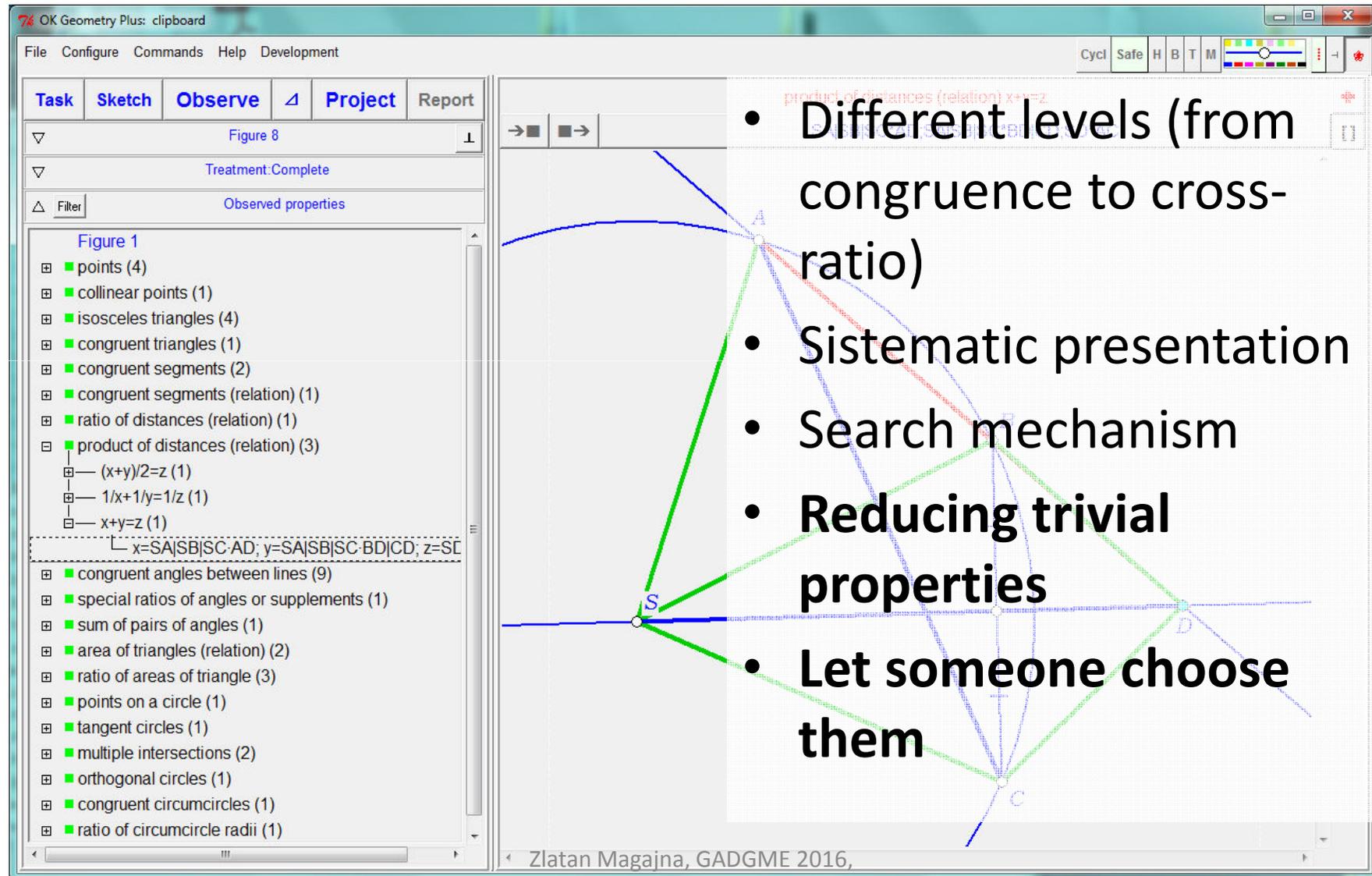
(\Leftarrow) Assume now that in the quadrilateral $ABCD$ the angles $\angle A$ and $\angle C$ are supplementary. We claim that $ABCD$ is cyclic.

Consider the circle k through A, B , and D . Assume, by contradiction, that C does not lay on k , but, for example, inside it. Let C' be the intersection, other than D , of the line BC with the circle k . Since $ABCD$ is cyclic, the angle $\angle C'$ is congruent to $\angle DCB$, since both are supplementary to $\angle A$. But this is impossible since in the triangle $\triangle C'CB$ the exterior angle at C cannot be congruent to the opposite interior angle.

Problems in automated observation - Tons of detected properties



Managing the observed properties



product of distances (relation) $x+y=z$

- Different levels (from congruence to cross-ratio)
- Systematic presentation
- Search mechanism
- **Reducing trivial properties**
- **Let someone choose them**

Figure 1

- points (4)
- collinear points (1)
- isosceles triangles (4)
- congruent triangles (1)
- congruent segments (2)
- congruent segments (relation) (1)
- ratio of distances (relation) (1)
- product of distances (relation) (3)
 - $(x+y)/2=z$ (1)
 - $1/x+1/y=1/z$ (1)
 - $x+y=z$ (1)

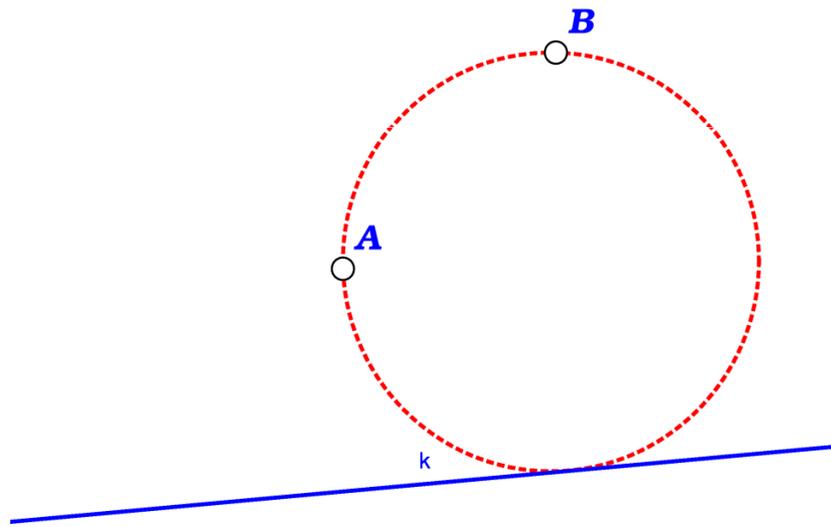
$x=SA|SB|SC \cdot AD; y=SA|SB|SC \cdot BD|CD; z=SE$

- congruent angles between lines (9)
- special ratios of angles or supplements (1)
- sum of pairs of angles (1)
- area of triangles (relation) (2)
- ratio of areas of triangle (3)
- points on a circle (1)
- tangent circles (1)
- multiple intersections (2)
- orthogonal circles (1)
- congruent circumcircles (1)
- ratio of circumcircle radii (1)

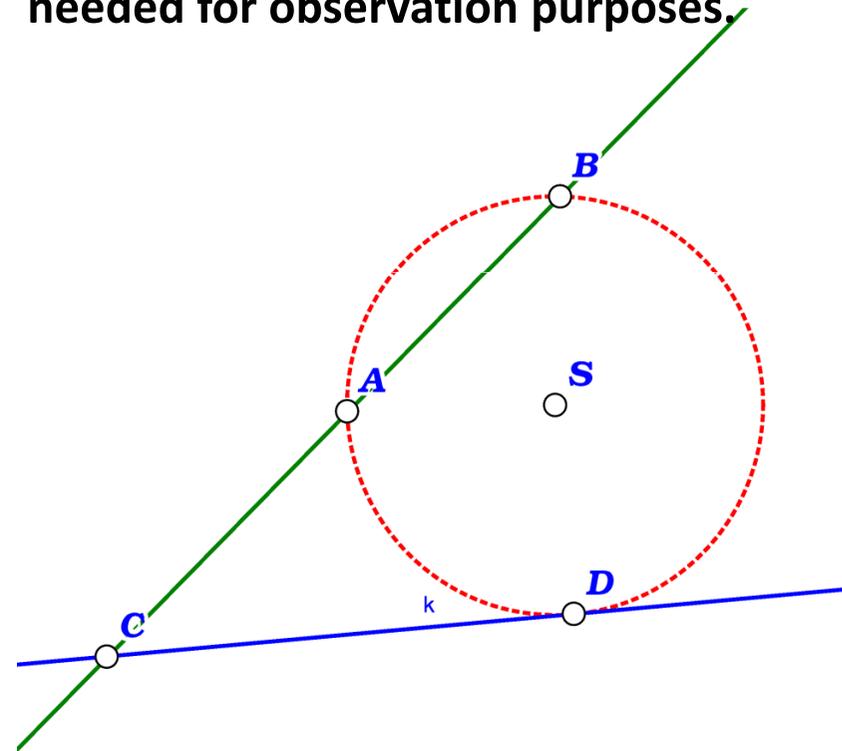
Zlatan Magajna, GADGME 2016,

Problems in observation – difficult objects

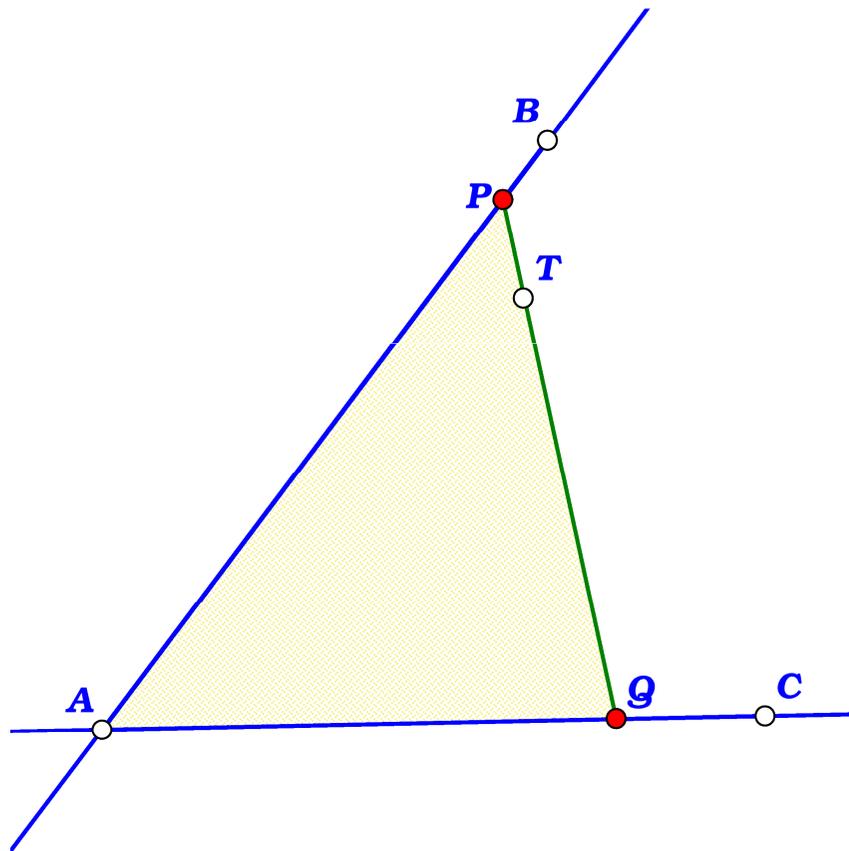
A circle passing through two given points and touching a given line.



Difficult objects are objects that one is unable to construct. Yet, the object is needed for observation purposes.



Difficult and new objects

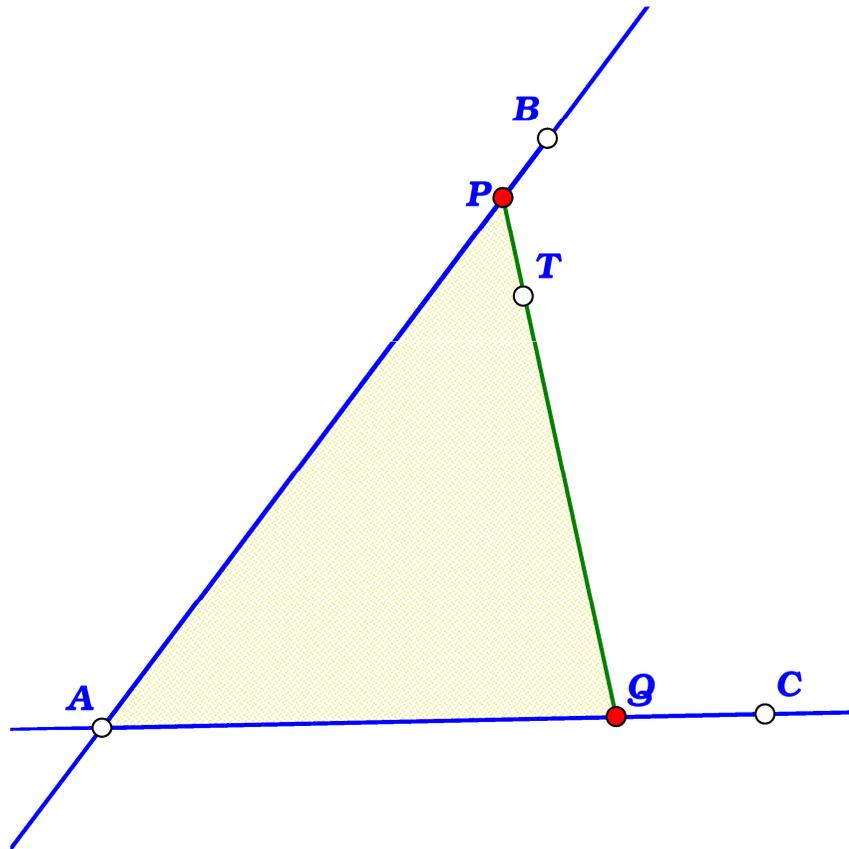


Given are two lines, AB and AC, and a point T. Find the shortest line segment PQ passing through T with P on AB and Q on AC.

PQ is a difficult object.

Difficult and new objects

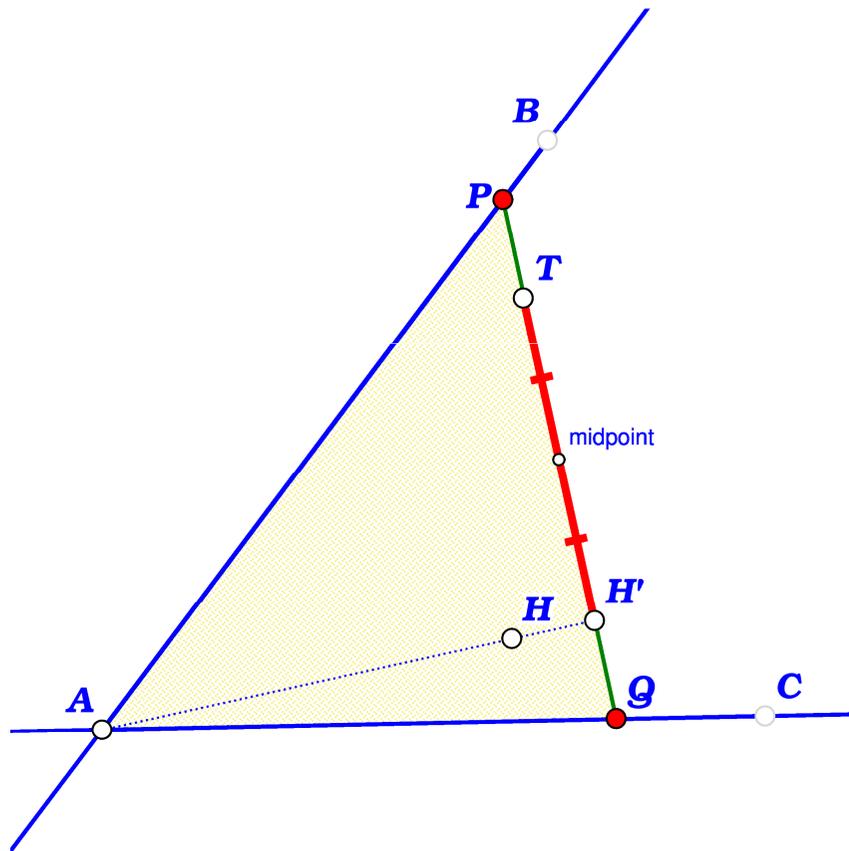
OK Geometry relates T to triangle ABC:



Consider the projection to the baseline containing T of these APQ related points: (35 items, great caution)

- X69: SYMMEDIAN POINT OF THE ANTICOMPLEMENTARY TRIANGLE (central projection from vertex to T)
- X4: ORTHOCENTER (central projection from vertex to mirror image of T wrp to side bisector)
- X25: HOMOTHETIC CENTER OF ORTHIC AND TANGENTIAL TRIANGLES (central projection from vertex to mirror image of T wrp to angle bisector)
- X20: DE LONGCHAMPS POINT (orthogonal projection to baseline to T)
- U043: 2nd ACACIA POINT (U43) (median parallel projection to midpoint of T and a triangle vertex)

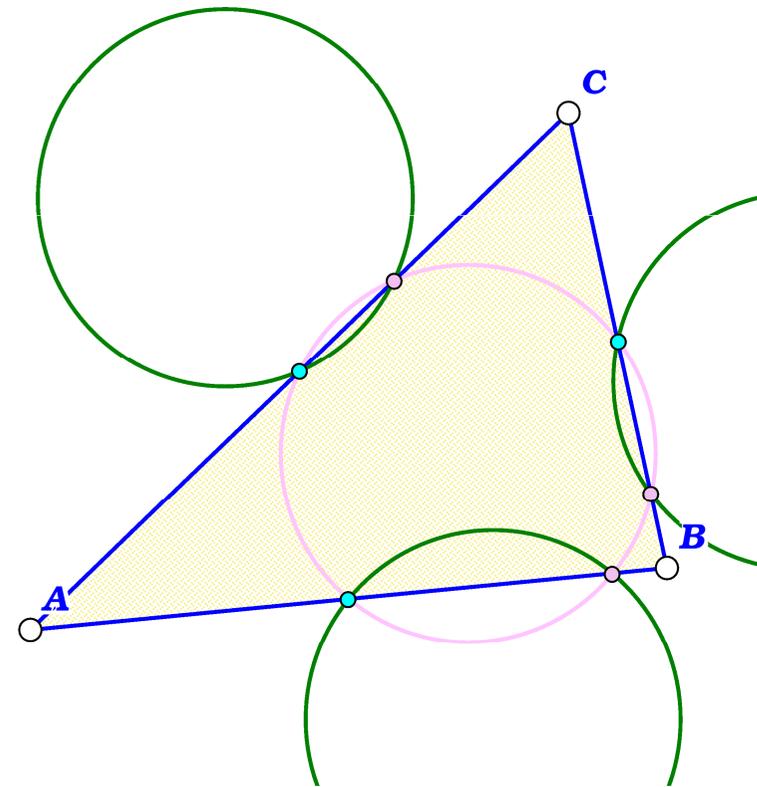
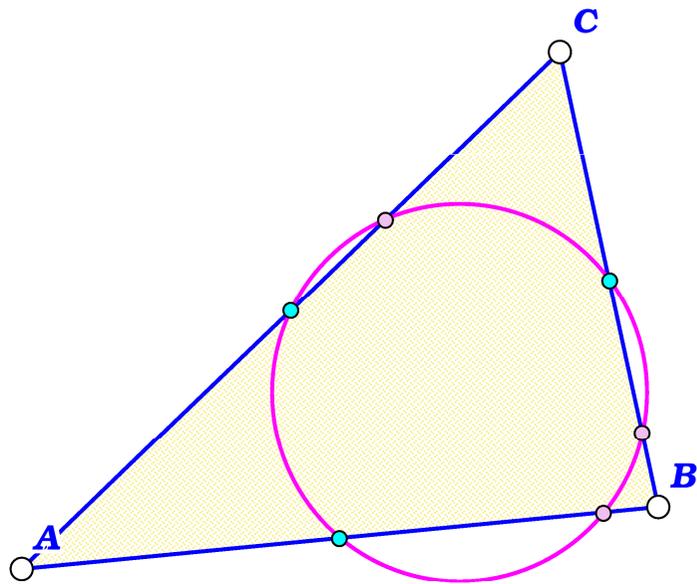
Difficult and new objects



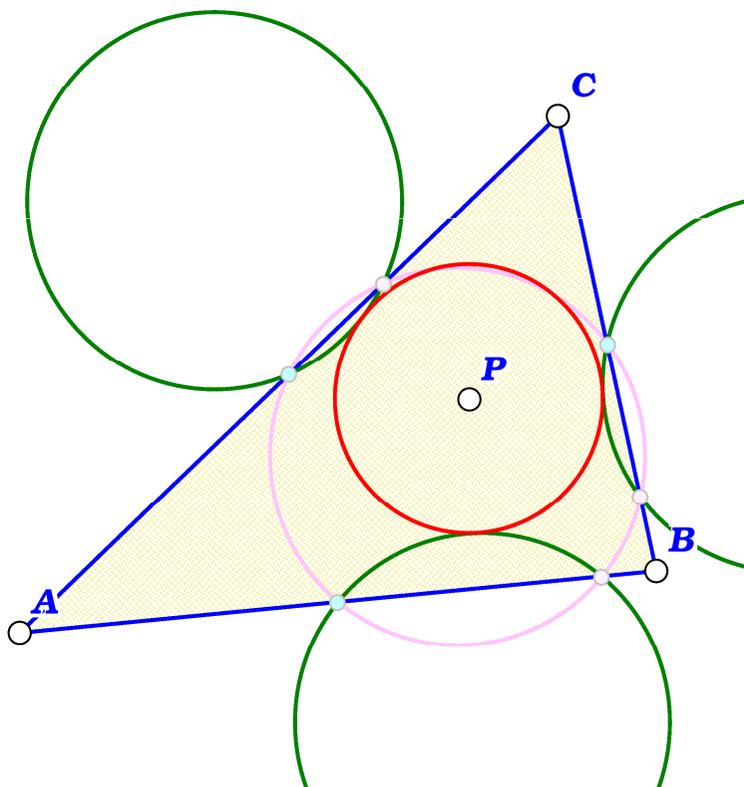
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- U043: 2nd ACACIA POINT (U43) (median parallel projection to midpoint of T and a triangle vertex)

The 9-point circle of a triangle is mirrored in each triangle's side.



A circle fits between the mirrored circles. What point is its centre P?



Triangle centre analysis of P Reference triangle: ABC

Consider centres X1 - X9999 More Extensive Continue

The point P contains these ABC related finite points: (1 items, reliable)
 X_{54} :KOSNITA POINT

Transformed P matches these ABC related points: (2 items, reliable)
 P = Isogonal conjugate(X_5 :NINE-POINT CENTER)
 P = X_{54} :KOSNITA POINT

The point P touches these lines of the triangles related to ABC: (1 items, reliable)
 Euler Line of Circumorthic triangle

The point P is the midpoint of these centres in triangle ABC: (8 items, reliable)
 X_1 :INCENTER and X_{9905} :ORTHOLOGIC CENTER OF THESE TRIANGLES
 X_{1263} :ISOGONAL CONJUGATE OF X(1157) and X_{6343} :HATZIPOLYCENTRE
 X_3 :CIRCUMCENTER and X_{195} :X(5)-CEVA CONJUGATE OF X(3)
 X_{6276} :OUTER-GREBE-TRIANGLE-ORTHOLOGIC CENTER OF THESE TRIANGLES
 ...

Inversion analysis of P in ABC related circles: (1 items, reliable)
 P = Inverse in Circumcircle of X_{1157} :INVERSE-IN-CIRCUMCIRCLE

(Possibly transformed) P lays on these ABC related conics: (1 items, reliable)
 P on Jerebak hyperbola

The point P lays on the these conics through vertices of ABC and ...
 (only X_1 - X_{31} are considered): (6 items, quite reliable)

Conic:
 X_2 :CENTROID
 X_{24} :PERSPECTOR OF ABC AND ORTHIC-OF-ORTHIC TRIANGLE

Conic:
 X_3 :CIRCUMCENTER
 X_4 :ORTHOCENTER

Additional explanation of P

Description

Let O be the circumcenter of a given triangle ABC , and D the circumcenter of triangle BOC . Define E and F cyclically. The **Kosnita point** of ABC is the point of concurrence of lines AD , BE , and CF . The Kosnita point is the Kimberling centre $X(54)$.

References

Douillet, L. "Translation of the Kimberling's Glossary into barycentrics"
<http://www.douillet.info/~douillet/triangle/glossary/glossary.pdf>.

Kimberling, C. "Encyclopedia of Triangle Centers."
<http://faculty.evansville.edu/ck6/encyclopedia/>.

Kimberling, C. "Triangle Centers."
<http://faculty.evansville.edu/ck6/tcenters/>.

Weisstein, E. "Mathworld"
<http://mathworld.wolfram.com/>
<http://faculty.evansville.edu/ck6/encyclopedia/BicentricPairs.html>

OK

The diagram shows a triangle ABC with vertices A , B , and C . The circumcenter O is marked with a green dot. Three circles are drawn: a large blue circle centered at O passing through A , B , and C ; a smaller blue circle centered at D passing through B and C ; and two smaller blue circles centered at E and F passing through A and C , and A and B respectively. Dashed magenta lines AD , BE , and CF are drawn, intersecting at the red dot X , which is the Kosnita point. A vertical dashed magenta line also passes through X and O .

Continue

1 items, reliable)

tems, reliable)
(TER)

to ABC: (1 items,

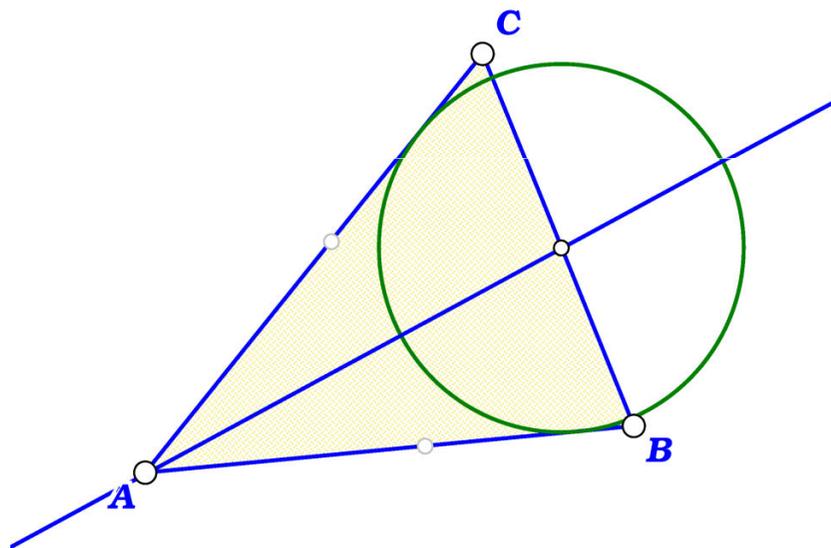
ABC: (8 items, rel
 TER OF THESE T
 and X_{6343} :HATZIP
 ONJUGATE OF $X(3$
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s, reliable)
 E-IN-CIRCUMCIRC
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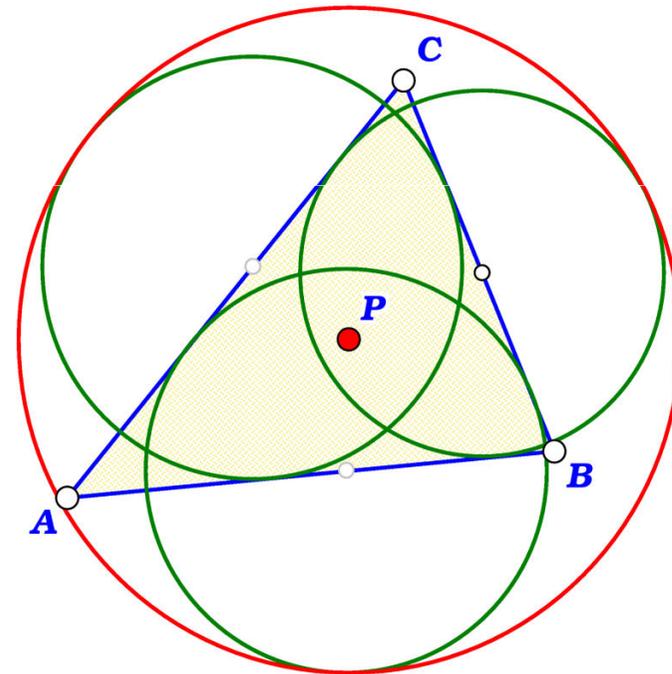
of ABC and ...

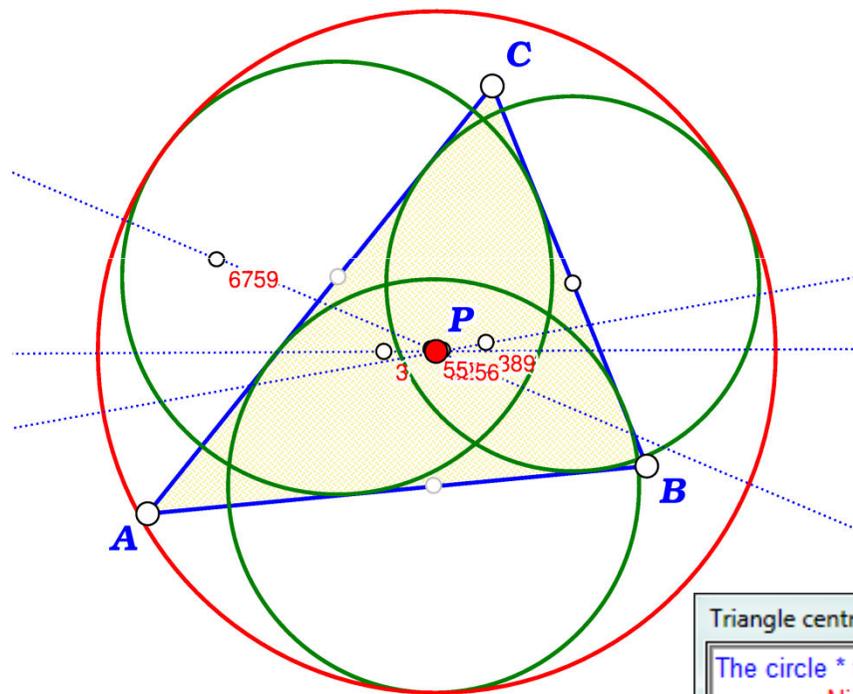
ORTHIC-OF-ORTH

In a triangle ABC consider the circles with center on one side and touching the other two sides.



Consider the circle touching the three circles from outside. What can be said about this circle and its center?





Triangle centre analysis of P Reference triangle: ABC

Consider centres X1 - X9999 More Extensive Continue

The point P lays on lines through these finite centres of ABC: (8 items, q)

Line: X₁:INCENTER
X₇₀₆₆:PERSPECTOR OF ABC AND THE EXTRA-TRIANGLE

Line: X₃:CIRCUMCENTER
X₂₂₅₆:X(2)-ISOCONJUGATE OF X(937)

Line: X₃₇:CROSSPOINT OF INCENTER AND CENTROID
X₃₈₉:CENTER OF THE TAYLOR CIRCLE

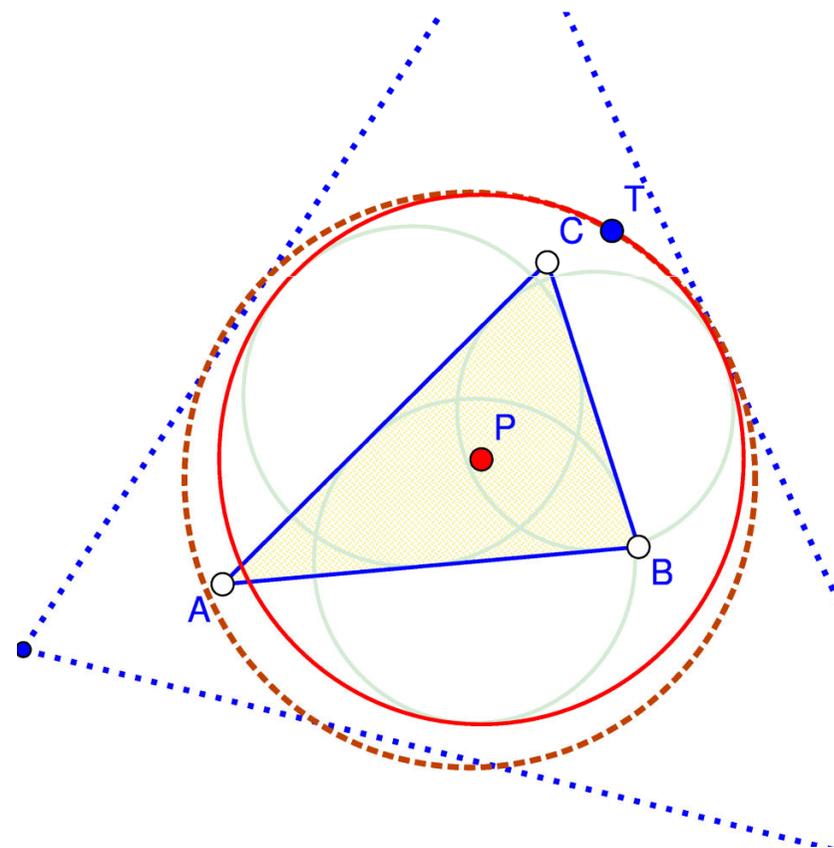
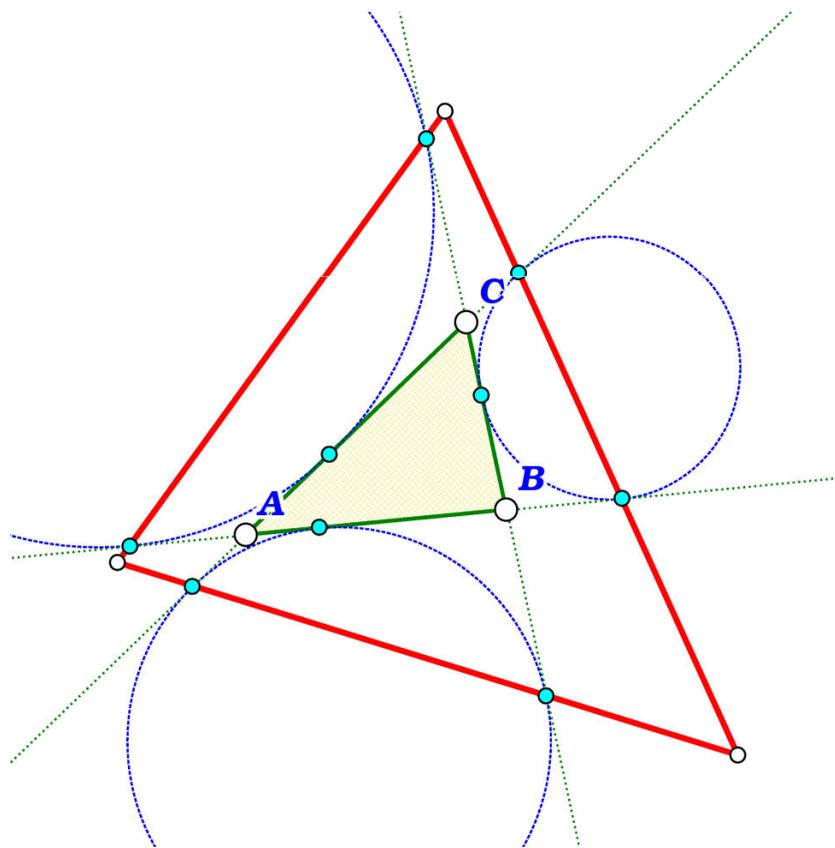
Line: X₅₅:INSIMILICENTER(CIRCUMCIRCLE, INCIRCLE)
X₆₇₅₉:INTOUCH-TO-ABC FUNCTIONAL IMAGE OF X(1)

Triangle centre analysis of * Reference triangle: ABC

The circle * touches these circles of the triangles related to ABC: (1 item)

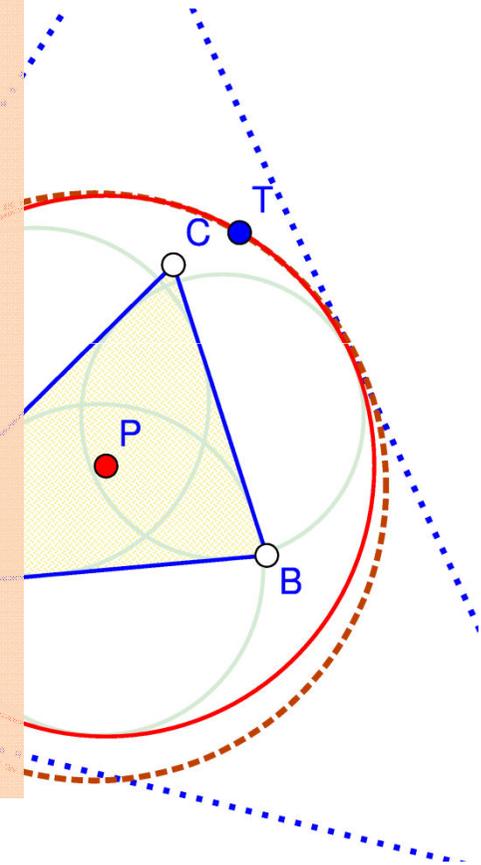
Nine point circle of 2nd extouch triangle

2nd excentral triangle



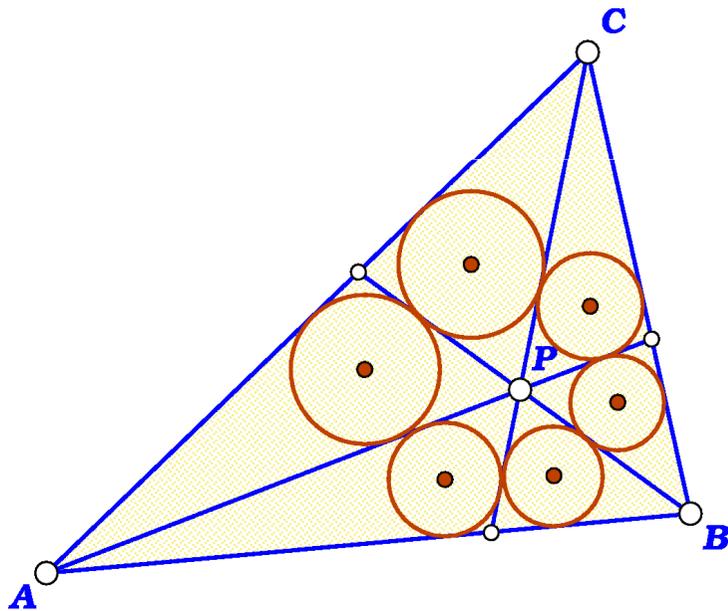
In the triangle observation OK Geometry considers

- 10 000 triangle centres (-> geometric meanings)
- Millions of lines through these centres
- ~50 geometric operations among the centres
- ~400 conics related to the reference triangle
- ~90 triangles related to the reference triangle
- ~10 lines related to each of these triangles
- ~30 circles related to each of these triangles

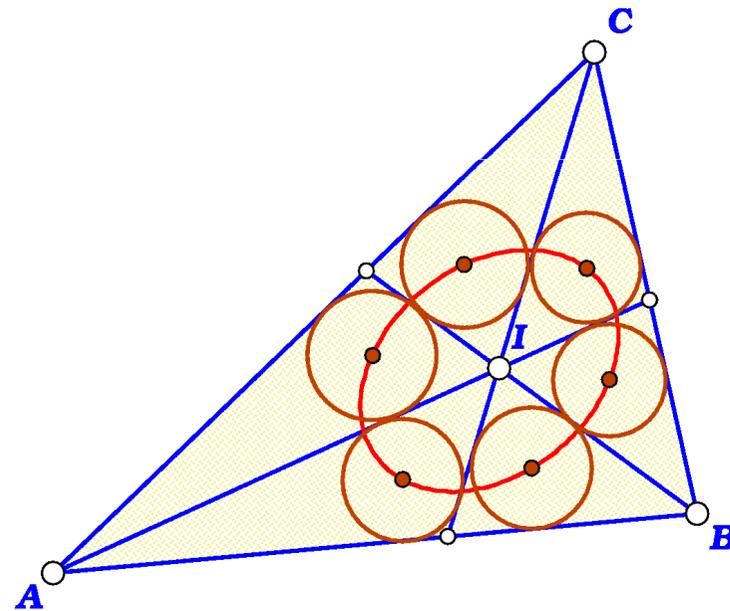


2nd excentral triangle

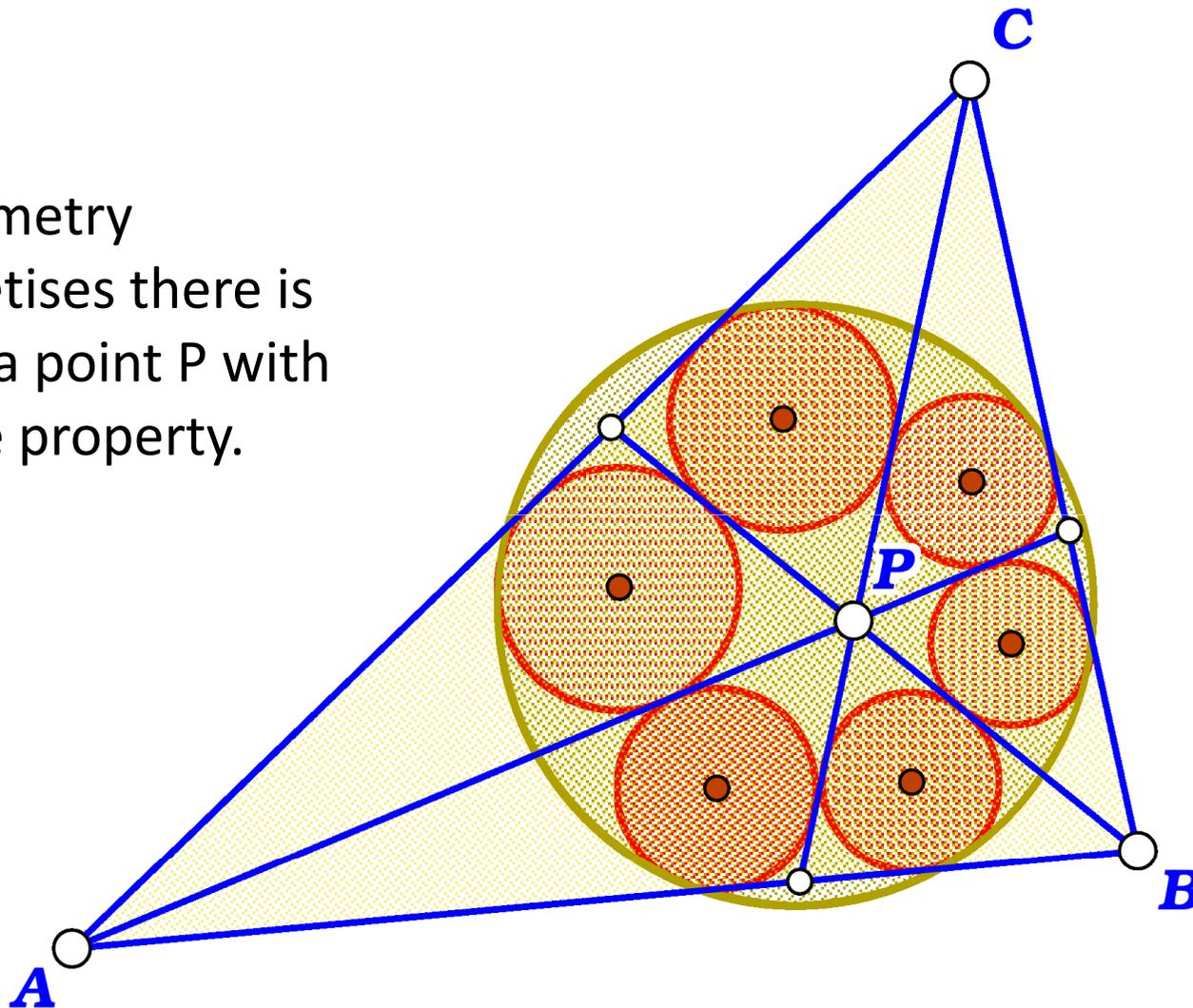
A point P in a triangle gives rise to 6 small triangles. The incentres of these triangles, in general, do not lay on a same conic.



The incentres lay on the same conic if, for example, P is the incentre of ABC .



OK Geometry
hypothesises there is
always a point P with
this nice property.



Technology and writing up proofs

- Various dimensions of comprehending proofs (Weber, Selden,...)
- Various levels of comprehending proofs (Lin)
- Various presentation modes (Herbst, Wong,...)
- Various specific tools
- There is not a single good way of presenting a proof.

Technology and writing up proofs

- Various dimensions of comprehending proofs (Weber, Selden,...)
- Various levels of comprehending proofs (Lin)
- Various presentation modes (Herbst, Wong,...)
- Various specific tools
- Meaning of terms and statements
- Justification of claims
- Logical structure (proof framework)
- High level ideas (structure of proof)
- General method
- Application of proof

Justification of claims

The image shows a screenshot of a geometry software interface with a sequence of five diagrams illustrating the justification of claims for an angle bisector in a circle. The diagrams are numbered 1 through 5.

Diagram 1: Shows a circle with center S and a secant line AD passing through the circle. Points A , B , and C are on the circle. A line BC is drawn. A line SD is drawn from the center S to point D . A green shaded angle $\angle CSD$ and a red shaded angle $\angle CAD$ are shown. The property $\angle CSD = \angle CAD$ is displayed in the "Property" field of the "Icon editor" window. The "Comment" field is empty.

Diagram 2: Shows the same setup as Diagram 1, but with a red dashed circle passing through points A , S , and C . The text below the diagram asks: "Point D lays on the circle through A,S,C. Why?"

Diagram 3: Shows the same setup as Diagram 1, but with a green shaded angle $\angle DSB$ instead of $\angle CSD$. The text below the diagram asks: " $\angle CSD = \angle DSB$. Why?"

Diagram 4: Shows the same setup as Diagram 1, but with a red shaded angle $\angle CAD$ instead of $\angle CSD$. The text below the diagram asks: " $\angle CSD = \angle CAD$. Why?"

Diagram 5: Shows the same setup as Diagram 1, but with a blue shaded angle $\angle CSB$ instead of $\angle CSD$. The text below the diagram asks: " $\angle CSB = 2 \cdot \angle CAB$. Why?"

The "Icon editor" window is open over the first diagram, showing the "Property" field with the text $\angle CSD = \angle CAD$ and the question "Why?". The "Comment" field is empty. The "Emphasize" button is selected. The "OK" and "Cancel" buttons are visible at the bottom of the window.

At the bottom of the software interface, the text "Zlatan Magajna, GADGME 2016, TarguMures" is visible.

High-level ideas

OK Geometry Plus: Miquel_hyper_proof.pro

File Configure Commands Help

Task Sketch Observe **Project** Report

Miquel theorem

Given is a triangle $\triangle ABC$. Let A' , B' , and C' be arbitrary points on the sides BC , CA , and AB . The three circles through A, B', C' , through A', B, C' , and through A', B', C always meet in a common point.

Observed properties

- Miquel theorem**
Given is a triangle $\triangle ABC$. Let A' , B' , and C' be arbitrary points on the sides BC , CA , and AB . The three circles through A, B', C' , through A', B, C' , and through A', B', C always meet in a common point.
- Strategy of the proof**
Let P be the intersection other than A' of the circles through B, C, A' and through C, A, B' . We shall prove that P lays on the circle through A, B', C' .
- Idea of the proof**
We shall prove that A, C, P, B' are cocyclic, i.e. that $ACPB'$ is a cyclic quadrilateral. To prove this we shall use the theorem:
A quadrilateral is cyclic if and only if its non adjacent angles are supplementary.
 - Theorem**
A quadrilateral $ABCD$ is cyclic if and only if its non-adjacent angles are supplementary.
 - Proof \Rightarrow**
(\Rightarrow) Let $ABCD$ be a cyclic quadrilateral. Thus $ABCD$ is inscribed in a circle, let its centre be S . Consider the opposite angles $\angle A$ and $\angle C$. These angles are related to complementary arcs $B\hat{C}D$ and $D\hat{A}B$. The

Icon size Emphasize Bleach Highlight Labels Angles Other

1 Miquel theorem

2 Strategy of the proof

3 Idea of the proof

4 Proof

Technology and writing up proofs

- Various dimensions of comprehending proofs (Weber, Selden,...)
 - Various levels of comprehending proofs (Yang, Lin)
 - Various presentation modes (Herbst, Wong,...)
 - Various specific tools
- Surface
 - Recognising elements
 - Chaining elements
 - Encapsulation
- (Scaffolding at various levels)

Chaining elements

OK Geometry Plus: Proof1.pro

File Configure Commands Help Development

Task Sketch Observe \triangle Project Report

Observed properties

1 Task
 Given is a circle with centre S and three points, A, B, C on its circumference. Let D be the intersection of the line AB and the bisector of the chord BC. Prove that D lies on the circle through A, S, C.

2 Point D lies on the circle through A,S,C. Why?
 3 $\angle CSD = \angle DSB$. Why?
 4 $\angle CSD = \angle CAD$. Why?
 5 $\angle CSB = 2 \angle CAB$. Why?

Icon size Emphasize Bleach Highlight Labels Angles

1 Task

2 Point D lies on the circle through A,S,C. Why?

3 $\angle CSD = \angle DSB$. Why?

4 $\angle CSD = \angle CAD$. Why?

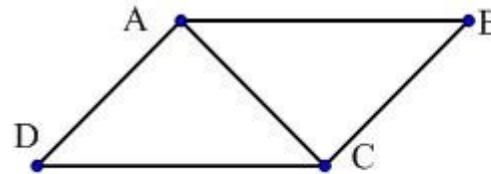
5 $\angle CSB = 2 \angle CAB$. Why?

Technology and writing up proofs

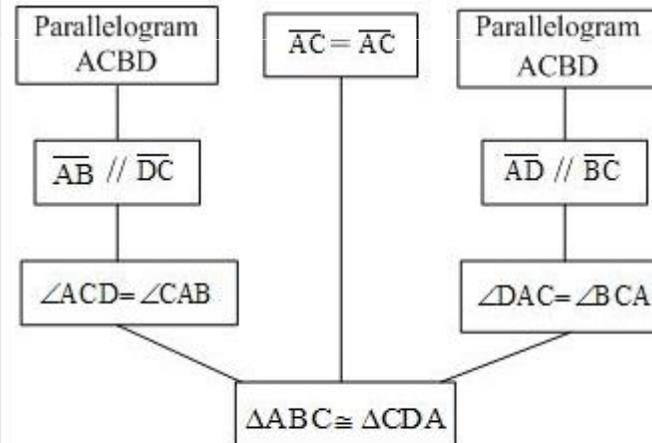
- Various dimensions of comprehending proofs (Weber, Selden,...)
- Various levels of comprehending proofs (Lin)
- Various presentation modes (Herbst, Wong,...)
- Various specific tools
- Figure, dynamic figure
- Flowchart
- Two column mode
- Paragraph mode

Multiple representations – Mr Geo (Wong, Yin, Yang, Cheng, 2011)

Given: Parallelogram ABCD
with diagonal \overline{AC}
Prove: $\triangle ABC \cong \triangle CDA$



1. ABCD is a parallelogram (Given)
2. \therefore ABCD is a parallelogram, $\therefore \overline{AB} \parallel \overline{DC}$
(Def. of parallelogram)
3. \therefore ABCD is a parallelogram, $\therefore \overline{AD} \parallel \overline{BC}$
(Def. of parallelogram)
4. $\therefore \overline{AB} \parallel \overline{DC}$, $\therefore \angle ACD = \angle CAB$ (Alt. int. angles)
5. $\therefore \overline{AD} \parallel \overline{BC}$, $\therefore \angle DAC = \angle BCA$ (Alt. int. angles)
6. $\overline{AC} = \overline{AC}$ (Reflexive law)
7. $\triangle ABC \cong \triangle CDA$ (ASA)



Technology and writing up proofs

- Various dimensions of comprehending proofs (Weber, Selden,...)
- Various levels of comprehending proofs (Lin)
- Various presentation modes (Herbst, Wong,...)
- **Various specific tools**

A D H statements

OK Geometry Plus: Proof2.p

File Configure Commands Help

Task Sketch Observe **Project** Report

Task

D: Let E be the midpoint of BC.

H1: $\angle CSB = 2 \cdot \angle CSD$

A: First, note that S lays on the bisector of segment BC (since $|SB|=|SC|$). Let E be the midpoint of BC. The triangles AEB and SEC are congruent by sss. Thus

$$\angle CSE = \angle ESB$$

and consequently

$$\angle CSB = 2 \cdot \angle CSD.$$

H2: $\angle CAB = \angle CSD$

A: The arc BC of the circle $k(S,A)$ spans an inscribed angle $\angle CAB$ and the central angle $\angle CSB$. By a known theorem the inscribed angle $\angle CAB$ measures one half of the central angle $\angle CSA$ over the same arc. On the other hand $\angle CSD$ is also one half of the $\angle CSB$ as shown above.

H3: The points A, S, C, D are cocyclic.

A: Consider the circle passing through S, C, D and its arc between C and D. Note that S and A lay on the same side of the line CD. It is known that the points X for which $\angle CXD = \angle CSD$ lay on the complementary arc of CD in circle through S, C, D. Since $\angle CAB = \angle CSD$ the points A, S, C, D are cocyclic.

Emphasize Bleach Highlight Labels Angles Steps Other

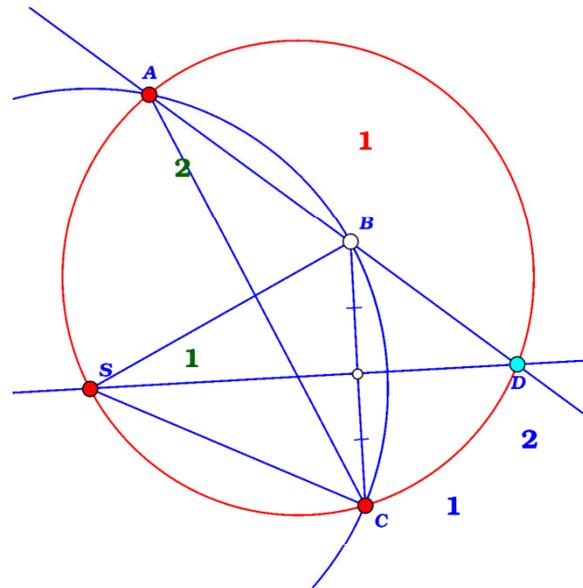
Task

1 Task

Given is a circle with centre S and three points A, B, C on its circumference. Let D be the intersection of the line AB and the bisector of the chord BC .

Prove that $S, C, D,$ and A are cocyclic.

Comment:



2 Proof

Definition Let E be the midpoint of BC .

Claim 1 $\angle CSB = 2 \cdot \angle CSD$

Argument 1 First, note that S lies on the bisector of segment BC (since $|SB|=|SC|$). Let E be the midpoint of BC . The triangles AEB and SEC are congruent by sss. Thus

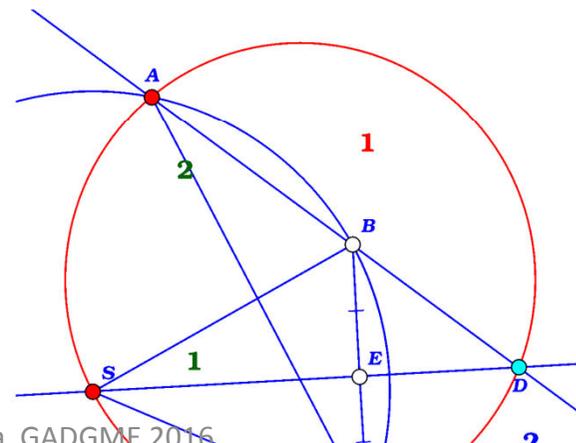
$$\angle CSE = \angle ESB$$

and consequently

$$\angle CSB = 2 \cdot \angle CSD.$$

Claim 2 $\angle CAB = \angle CSD$

Argument 2 The arc BC of the circle $k(S,A)$ spans an inscribed angle $\angle CAB$ and the central angle $\angle CSB$. By a known theorem



Thank you!

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TarguMures

OK Geometry

<http://z-maga.si/index?action=article&id=40>